On the Temperature Field of a Square Column Embedding a Heating Cylinder

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An analysis of the two-dimensional temperature field of a square column embedding a heating cylinder is presented. The analysis leads to a solution in finite series which gives rise to rapidly converging numerical results. The series coefficients computed for the extreme case of the cylinder being a line find a way of parametrization, which then furnishes the solutions for finite cylinders up to the case of the cylinder diameter being half of the column thickness. Results in five-digit accuracy are presented. The method is simple and direct. In addition, it retains the same concise formal solution for a variety of boundary conditions.

This paper reports an analytical expression for the temperature field of a long square column in which a heating cylinder is embedded coaxially. It is assumed that the surface temperature of the cylinder is steady and uniform at t_a, and so is the wall temperature of the square column at t_b. Even though a rough solution to this problem can be obtained graphically (7), an analytical solution to this system has not yet been found in the literature.

METHOD OF SOLUTION

A polar coordinate system (r, θ) is employed with its origin placed at a corner, O in Figure 1. We introduce the dimensionless quantities

$$T = \frac{t - t_b}{t_a - t_b}, \quad R = \frac{r}{a} \tag{1}$$

into the governing Laplace's equation, so that it reads

$$\frac{\partial^2 T}{\partial R^2} + \frac{1}{R}\frac{\partial T}{\partial R} + \frac{1}{R^2}\frac{\partial^2 T}{\partial \theta^2} = 0 \tag{2}$$

Attention is paid herein to the particular situation where the radius of the heating cylinder r_c is infinitesimally small. As will be revealed soon, the solution to this particular case rather strikingly furnishes solutions to the cases $r_c >> 0$ up to $r_c \simeq 0.5a$ without solving them anew. The boundary conditions are therefore prescribed around $\triangle$OAB, Figure 2, as

$$\left.\begin{aligned}
&\text{a,}\quad T(R,0) = 0, \quad 0 \leq R \leq 1\\
&\text{b,}\quad \left.\frac{\partial T}{\partial \theta}\right|_{\theta=\pi/4} = 0, \quad 0 < R < \sqrt{2}\\
&\text{c,}\quad T\left(\sqrt{2}, \frac{\pi}{4}\right) = 1\\
&\text{d,}\quad \left|\frac{\partial T}{\partial R} - \sin\theta\left(\frac{\partial T}{\partial \theta}\right)\right|_{R=\sec\theta} = 0,\\
&\qquad\qquad 0 < \theta < \frac{\pi}{4}
\end{aligned}\right\} \tag{3}$$

where boundary conditions b and d each stipulates a Neumann condition along OA and BA.

The most general solution to Equation (2) may be obtained in the following form (5):

$$T(R,\theta) = (A_0 + B_0 \ln R)(C_0 + D_0\theta) + \sum_k (A_k R^k + B_k R^{-k})(E_k \cos k\theta + D_k \sin k\theta) \tag{4}$$

To fit Equation (4) to the present system, the $\ln R$ and the R^{-k} terms must be discarded lest T become infinite as R approaches zero. In order that T be single valued, D_0 must be zero. Boundary condition a then expels A_0 and E_k by demanding that T vanish at $\theta = 0$. The general solution is eventually stripped down to

$$T(R,\theta) = \sum_k A_k D_k R^k \sin k\theta \tag{5}$$

To comply with boundary condition b, one differentiates Equation (5) and requires $\cos(k\pi/4)$ be zero. This requirement is fulfilled by taking the eigenvalues to be

$$k = 2(2n-1), \quad n = 1, 2, 3, \ldots \tag{6}$$

Equation (5) states that $T(R, \theta)$, which satisfies Equation (2), along with boundary conditions a and b, may be constructed from any number of terms of the sequence $\{R^k \sin k\theta\}$ in a linear combination. Leaving the rationalizing arguments† elsewhere (9, 10), we adapt the first N terms and rewrite Equation (5) as

$$T_N^*(R,\theta) = \sum_{n=1}^{N} C_n^* R^{2(2n-1)} \sin 2(2n-1)\theta \tag{7}$$

where C_n^*'s have replaced the combined constants which are to be determined. The asterisks attached to $T_N(R, \theta)$ and C_n denote these solutions being specifical to the case under consideration, namely, the heating cylinder being considered as a line heat source.

Boundary condition c is then imposed on Equation (7) to obtain

$$\sum_{n=1}^{N} (-1)^{n+1} 2^{2n-1} C_n^* = 1 \tag{8}$$

In order to solve for the N unknown C_n^*'s, one must supplement $N - 1$ linear algebraic equations containing C_n^*'s to Equation (8). This set of equations may be obtained in one way or another in accordance with one's choice of an error distribution principle in the boundary methods (2, 3). Here, we follow the criterion of the collocation method (4), which allots $N - 1$ boundary points along boundary BA, and demand boundary condition d be satisfied at these selected points. Consequently, the N C_n^*'s are to be solved with Equation (8) and the set of $N - 1$ equations

$$\sum_{n=1}^{N} (2n-1) C_n^* R_\theta^{2(2n-1)} \sin(4n-3)\theta_i = 0,$$
$$i = 1, 2, 3, \ldots N-1, \quad 0 < \theta_i < \frac{\pi}{4} \tag{9}$$

where R_θ is the dimensionless radial coordinate of a boundary point along BA whose angular coordinate is θ_i. In this simple configuration, $R_\theta = \sec\theta_i$.

We allot the $N - 1$ boundary points (R_θ, θ_i) at equal

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† Being referred to as the rational approach, which led to the present scheme of employing cylindrical coordinates.

TABLE 1. FUNCTION COEFFICIENTS, C_n^*

			$N = 24$		
0.164749	-1.62237^{-2}	2.39648^{-3}	-4.27910^{-4}	8.32098^{-5}	-1.70198^{-5}
3.60351^{-6}	-7.79445^{-7}	1.71519^{-7}	-3.96132^{-8}	8.31266^{-9}	-1.71147^{-9}
9.89538^{-10}	3.68348^{-10}	3.43245^{-10}	1.62487^{-10}	6.77315^{-11}	1.71084^{-11}
2.34034^{-12}	-2.09752^{-13}	-1.94689^{-13}	-9.55035^{-14}	-3.01255^{-14}	-4.26090^{-15}
			$N = 12$		
0.191011	-1.88103^{-2}	2.77891^{-3}	-4.96376^{-4}	9.65998^{-5}	-1.97901^{-5}
4.19453^{-6}	-9.30120^{-7}	1.47280^{-7}	-1.35355^{-7}	-5.81467^{-8}	-2.28738^{-8}

(Note the abbreviation $\times$ 10 dropped from -1.62237×10^{-2}, etc.)

intervals. The solution was programmed on a computer with N as the parameter. Eleven values of N's taken from 2 up to 24 were employed for an investigation of the convergence. Two sets of the function coefficients C_n^* are given as examples in Table 1. For a quicker referral, some points in the field are labeled as indicated in Figure 2, and the $T_N^*(R, \theta)$ at a point p will be referred to as $T_N^*(p)$. One naturally expects the numerical values of $T_N^*(p)$'s to converge as N in Equation (7) is successively increased. Discouragingly, the rate of convergence turned out to be extremely slow. For instance, the two sets of C_n^* listed gave $T_{12}^*(f) = 0.14655$ and $T_{24}^*(f) = 0.12640$, respectively.

ANALYSIS

The dilemma was analyzed as follows. On the basis that the boundary condition around the neighborhood of point A has not been defined, it is postulated that Equation (8) might have diffused out from point A and in effect specified $T_N^*(A) = 1$ on a fictitious circle of radius r_δ. Assuming $0 < r_\delta < a/N$, we perceive that r_δ becomes smaller (but remains finite) as N is increased. This postulation should account for the apparently slow convergence of $T_N^*(R, \theta)$, because the shrinking of r_δ implies changing of a boundary condition of the system. We postulate further that the dimensionless temperature should be unique if r_δ is fixed (on the premise, naturally, that the series converges fast enough). It follows that the boundary condition $T(R, \theta) = 1$ should be prescribed on a fixed reference circle of radius r_p, and $r_\delta = r_p$ be demanded from all N's.

Before putting the preceding ideal into practice, we define, a priori, a dimensionless temperature $T(R, \theta)$ in analogy with Equation (7):

$$T(R, \theta) = \sum_{n=1}^{N} C_n R^{2(2n-1)} \sin 2(2n-1)\theta \qquad (10)$$

in which C_n is a new set of reduced function coefficients defined by

$$C_n = C_n^* / T_N^*(p) \qquad (11)$$

The relation

$$T(R, \theta) = T_N^*(R, \theta) / T_N^*(p) \qquad (12)$$

then follows readily from Equations (7), (10), and (11).

Inspecting Equation (12), one sees that $T(R, \theta)$ given by Equation (10) has incorporated into it the intrinsic property $T(p) = 1$ for all N's, because $T_N^*(p)$, by definition, is the $T_N^*(R, \theta)$ at point p predicted by Equation (7). Now suppose one chooses a convenient arbitrary point p on a would-be reference circle of radius r_p; he then uses the $T_N^*(p)$ as a reference temperature in reducing a corresponding set of N C_n's by means of Equation (11). The desired unique solution $T(R, \theta)$ is then given by Equation (10), which is no more dependent on N except for possible truncation errors incurred in smaller N's.

For each set of C_n^* in Table 1, fifteen reference temperatures taken along AB in Figure 2 are tabulated in Table 2. We are now able to demonstrate that $T(R, \theta)$ is unique. For example, when $T_{12}^*(5) = 0.324925$ and

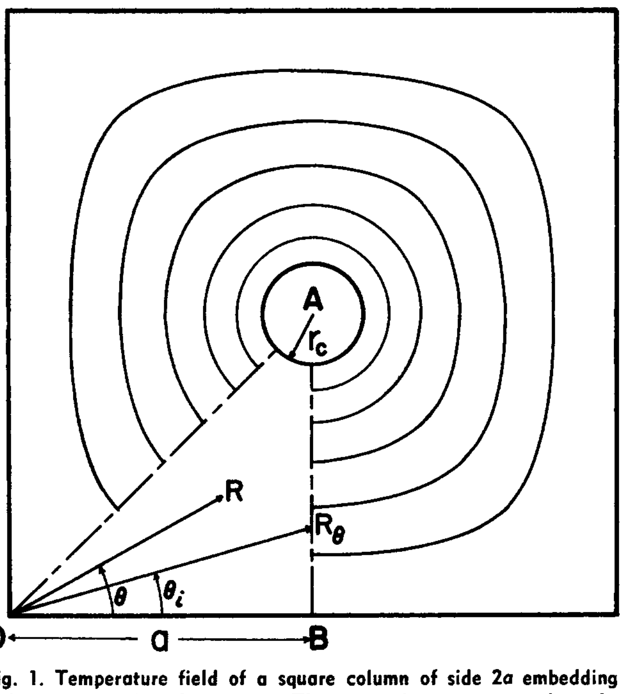

Fig. 1. Temperature field of a square column of side $2a$ embedding a heating cylinder of radius r_c. The triangular section to show the coordinate system in a repetitive segment of the field.

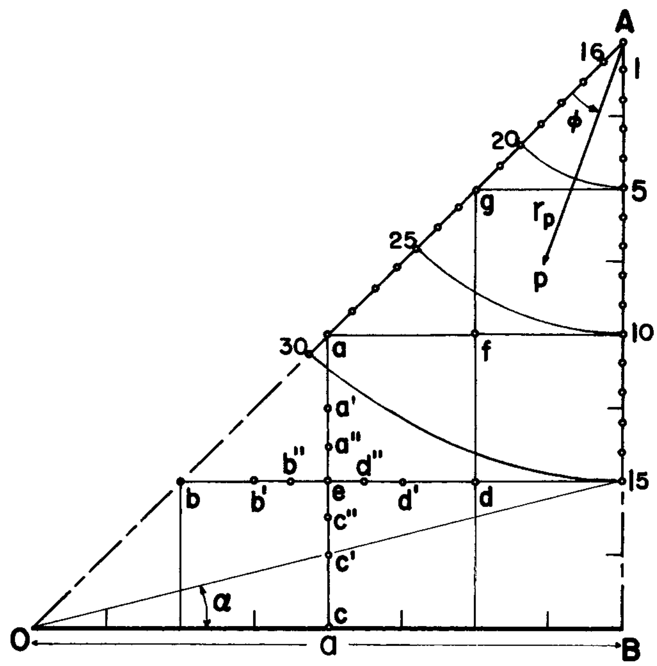

Fig. 2. Labeled reference points and representative grid points in a repetitive segment of the field.

TABLE 2. REFERENCE TEMPERATURES $T_N^*(p)$ ALONG AB AND TEMPERATURE RATIOS $T(p+15)/T(p)$

p	r_p/a	$T_{24}^*(p)$	$T_{12}^*(p)$	$T(p+15)/T(p)$
1	0.05	0.588708	0.696000	1.0000
2	0.10	0.455993	0.528279	1.0000
3	0.15	0.378183	0.437596	1.0001
4	0.20	0.323035	0.374689	1.0001
5	0.25	0.280224	0.324925	1.0004
6	0.30	0.245214	0.284252	1.0009
7	0.35	0.215565	0.249928	1.0020
8	0.40	0.189816	0.220080	1.0038
9	0.45	0.167019	0.193636	1.0070
10	0.50	0.146516	0.169869	1.0121
11	0.55	0.127832	0.148210	1.0203
12	0.60	0.110610	0.128240	1.0332
13	0.65	0.0945674	0.109641	1.0535
14	0.70	0.0794795	0.0921490	1.0855
15	0.75	0.0651579	0.0755441	1.1375

$T_{24}^*(5) = 0.280224$ are each employed as the reference temperatures, $T(f)$ is obtained as 0.45103 by $N = 12$, and as 0.45108 by $N = 24$.

Coming back to the argument of a reference circle, we draw arcs of radii r_p's across AO and AB and label the intersections on AO $p + 15$ in correspondence with the intersections p ($p = 1, 2, 3, \ldots 15$) on AB; for example, in Figure 2 point 20 is on the same arc as point 5. Since the temperature gradient along OA is the least steep, while that along BA is the steepest, each pair of $T(p + 15)$ and $T(p)$ for $p = 1$ to 15 is the upper and the lower bounds of the $T(R, \theta)$ along the connecting arc. As a consequence, the temperature ratio $T(p + 15)/T(p)$ must be unity should the connecting arc represent an isotherm. An examination of the temperature ratios listed in Table 2 reveals that the isotherms are indeed closely approximated by a family of concentric circles around point A. The maximum deviation at $r_p = 0.3a$ is but 0.1%, and if a deviation of 1.2% can be tolerated, the circle of r_p as large as $0.5a$ may still be deemed as an isotherm.

Because the isotherms around point A practically coincide with the reference circles, it is now apparent that the solutions to $r_c > 0$ up to $r_c \simeq 0.5a$ are already in hand. Selecting a reference temperature $T_N^*(p)$ at $r_p = r_c$, one reduces C_n from C_n^*. This particular set of C_n employed in Equation (10) yields $T(R, \theta)$, which is equivalent to the outcome of the problem in which the heating cylinder takes r_c as its radius. Of course, the approximate nature of such a solution should be recognized. The small deviations, which depend on r_c, may be estimated from the temperature ratios tabulated in Table 2. Incidentally, we have also thus shown that the proposition of reference circles is valid.

Schofield (8) had treated an analogous situation wherein the cylinder was embedded in an infinite wall. In his treatment, by the method of conformal mapping, Schofield replaced the heating cylinder with a line heat source so as to effect a conformal transformation. Although this replacement was an approximation, one of the coaxial cylindrical isotherms generated around the line heat source did effectively substitute the isothermal surface of a heating cylinder.

NUMERICAL RESULTS

The accuracy and the rate of convergence of the proposed series solution are herein briefly examined. Table 3 lists, for $N = 24$ and 12, seven dimensionless temperatures which have taken $T_N^*(5)$ as their reference. The computation having been carried out in eight-digit, floating-point arithmetic, the listed figures of $T(p)$ may be deemed effective to the last digit shown. The accuracy of $T(p)$ obtained by the set $N = 24$ is concluded to be five digits; those obtained by $N = 18$, 20, and 24 are all in agreement to the fifth digit. $T(p)$ obtained by the set $N = 12$ is then seen to be at least accurate to the fourth digit. The accuracy obtained with $N = 6$ was three digits or so. Thus, the method yields tolerable accuracy with moderate sized N. The accuracy increases with the increase in N, which in this method is due to the diminishing of the boundary-condition discretization error associated with Equation (9).

The N' which follows $N = 24$ in Table 3 denotes the number of terms required to yield the $T(p)$ immediately below within $\pm$ 1 unit of its last digit; for example, the first eight terms in the set of 24 C_n's will be required to yield $T(g)$ 0.76390 $\pm$ 0.00001. Analogously, the N' after $N = 12$ denotes the number of terms required to yield the $T(p)$ below within $\pm$ 1 unit of its fourth digit. $T(p)$'s in Table 3 are intentionally listed in a decreasing order of R. It is quite apparent that N' decreases almost exponentially as R decreases. The rate of convergence of $T(p)$ with respect to the number of terms in the truncated series is thus seen to be satisfactorily fast. One notices that $T(p)$ for R small converges particularly rapidly; only two to three terms are required to yield an asymptotic value with $R < 0.8$. The observed swiftness in convergence is due to a twofold factor, that the orders of magnitude of C_n's decrease as n is increased, and that R is raised to the power of $2(2n - 1)$ in Equation (10).

In Figure 3, $T(p)$ vs. r_p along AB has been plotted in solid curve and that along AO in broken curve, with r_c as the parameter. The region where the solid curves coincide with the broken curves is one where the isotherms are essentially concentric circles around point A. The isotherms for a representative case of $r_c = 0.25a$ are mapped in Figure 4. It is interesting to compare the temperature field of Figure 4 with that of an unsteady state conduction of heat from a square column initially at a uniform temperature (1a). Either field bears nearly circular isotherms. But the gradients of isotherms along OA and BA with the field of Figure 4 are both steadily increasing, while the other has an S shaped gradient along OA and a decreasing gradient along BA.

The most valuable outcome of the present solution is perhaps its analytic form. Consider the rate of heat loss through the four sides of the wall per unit length of the column section:

$$q = 8 \int_0^a k_t \left(\frac{\partial t}{\partial \theta} \right)_{\theta=0} dr \tag{13}$$

Rewriting Equation (13) into

TABLE 3. DIMENSIONLESS TEMPERATURE $T(p)$ AT SELECTED POINTS FOR AN EXAMINATION OF ACCURACY AND CONVERGENCE

p	5	g	f	d	a	e	b
R	1.25	1.06	0.901	0.790	0.707	0.560	0.472
$N = 24, N'$	20	8	5	3	3	3	2
$T(R, \theta)$	1.00000	0.76390	0.45108	0.20723	0.30148	0.14633	0.073603
$N = 12, N'$	10	7	4	3	3	2	2
$T(R, \theta)$	1.00003	0.76383	0.45103	0.20721	0.30145	0.14632	0.073596

$$q = 8a\,(t_a - t_b) \int_0^1 k_t \left(\frac{\partial T}{\partial \theta} \right)_{\theta=0} dR \qquad (13a)$$

and assuming k_t to be constant, we obtain

$$q = 16ak_t\,(t_a - t_b) \sum_{n=1}^{N} \frac{2n-1}{4n-1} C_n \qquad (14)$$

Convergence of q is very rapid; no more than the first five C_n's are required to obtain a solution accurate to the fourth digit. For $r_c = 0.25a$, Equation (14) gives $q = 2.79123ak_t\ (t_a - t_b)$.

The dimensionless rate of heat loss is plotted in Figure 5 as a function of the dimensionless radius of the heating cylinder. Notice that Figure 5 correlates all six variables involved in a dimensionless form. It is, therefore, a simple matter to respond to a supposition such as: given q, a, k_t, t_b, and r_c/a, solve for t_a, etc.

DISCUSSIONS

Of theoretical interest in connection with the present system is the limiting situation where $r_\delta \to 0$. In rectangular Cartesian coordinates with the origin located at O, the temperature field of Figure 1 for this case must satisfy

$$\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} = -\,\delta(X-1)\,\delta(Y-1) \qquad (15)$$

The Green's function satisfying the above Poisson's equation with the homogeneous boundary conditions $T = 0$ on all four boundary lines may be derived from that given by Morse and Feshbach (*6*) or by integrating that given by Carslaw and Jaeger (*1b*) over τ from zero to t and then taking the limit as $t \to \infty$. It is

$$T(X, Y/1, 1) = \frac{4}{\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{m^2+n^2} \sin\frac{m\pi X}{2} \sin\frac{n\pi Y}{2} \sin\frac{m\pi}{2} \sin\frac{n\pi}{2} \qquad (16)$$

The elegant expression of Equation (16), however, gives numerical solution of poor convergence.

For a finite but very small r_δ, the present method requires estimation of the length of r_δ in order that $T(r_\delta) = 1$ may be taken as the reference temperature. r_δ as a function of N is estimated as follows. A reference point along AB is arbitrarily chosen at 10 in Figure 2. $T(10)$ vs. r_c taken along $r_p = 0.5a$ is plotted by making use of Figure 3. r_δ is then located on the abscissa by intersecting a horizontal line drawn from the ordinate at $T_N^*(10)$ with the extrapolated plot as shown in Figure 6. For instance, $T_{24}^*(10) = 0.1465$, and r_δ is estimated as $0.008a$. The rationale of the present estimation is clear on inspecting Equation (11) and Figure 3.

In a case where the radius of the heating cylinder is so large that its periphery can no longer be considered as coinciding with an isotherm, the problem could be handled by modifying boundary conditions c and d. As an illustration, for $r_c = 0.75a$ shown in Figure 2, boundary condition d of Equation (3) applies to $0 < \theta < \alpha$. Boundary condition c is substituted by

$$T(R_\theta, \theta) = 1, \quad \alpha \leq \theta \leq \pi/4 \qquad (17)$$

wherein

$$R_\theta^2 = 2 + \left(\frac{r_c}{a}\right)^2 - 2\sqrt{2}\left(\frac{r_c}{a}\right)\cos\phi \qquad (18)$$

and

$$\theta = \frac{\pi}{4} - \sin^{-1}\left(\frac{r_c}{aR_\theta}\sin\phi\right) \qquad (19)$$

relates θ and ϕ for $\alpha \leq \theta \leq \pi/4$. The R_θ in Equation (18) is paired to θ in Equation (19) by eliminating the parameter ϕ. One then allocates L boundary points along arc 15-30 and M boundary points along line B-15 to solve for the N C_n's in Equation (10), where $N = L + M$.

There are three extensions of the preceding case to which the present method of solution could apply. The first is a case whose boundary condition along arc 15-30 is a prescribed function, say $T(R_\theta, \theta) = f(\theta)$ for $\alpha \leq \theta \leq \pi/4$. The second is one where $T(R_\theta, \theta) = 1$ for $\alpha \leq \theta \leq \pi/4$ holds, but boundary 15-30 is a prescribed curve instead of an arc, say $R_\theta = g(\theta)$. The third is a combination of the two cases; for example, $T(R_\theta, \theta) = f(\theta)$ wherein $R_\theta = g(\theta)$. So long as $f(\theta)$ and $g(\theta)$ are single valued functions of θ, they may only be sectionally smooth, or even in tabulated forms. Equations (10) and (14) preserve their forms for all such modifications in boundary conditions; the apparent change will be in the numerical values of C_n's.

It appears desirable to compare the accuracy of the present method with that of the more well-known finite-difference method. Suppose one writes a typical difference equation for the network *abcd* and *e* of Figure 2:

$$T(a) + T(b) + T(c) + T(d) - 4T(e) = 0 \quad (20)$$

Let $T_r(e)$ be the actual numerical value of the left-hand side of Equation (20), in which the values of $T(p)$ com-

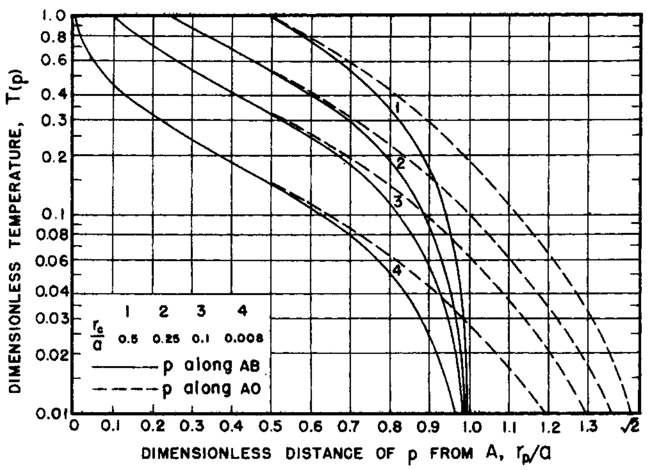

Fig. 3. Temperature—cylinder-radius relation as functions of positions along AB and AO of Figure 2.

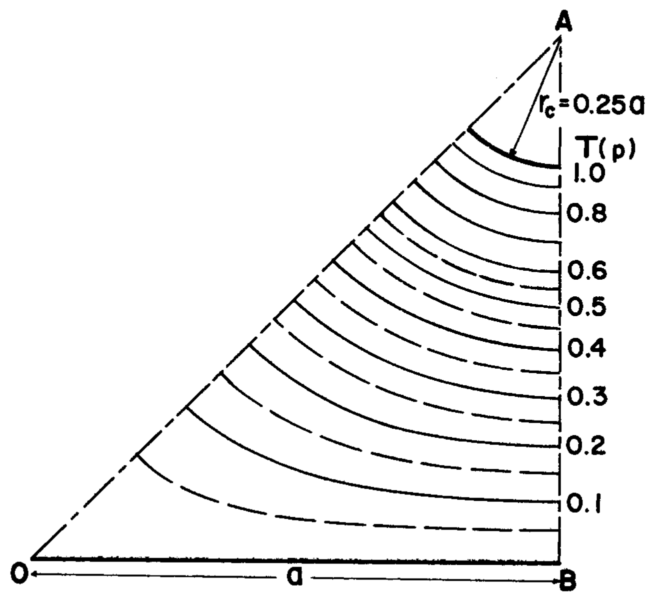

Fig. 4. Isotherms with heating cylinder radius 0.25a for a repetitive segment of the field.

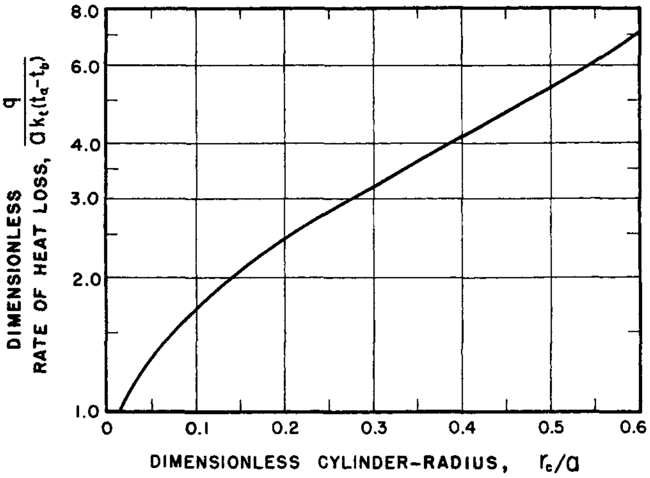

Fig. 5. Dimensionless rate of heat loss as a function of the dimensionless cylinder radius.

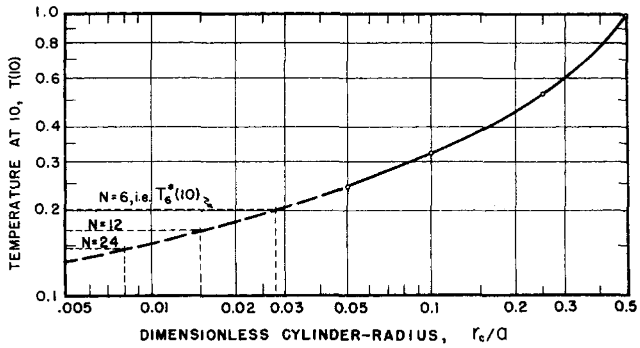

Fig. 6. Dimensionless temperature at point 10 as a function of the dimensionless cylinder radius for estimating r_δ.

puted by the present method for $p = a, b, c, d$, and e have been substituted. We define $T_r(e)$ the residual temperature at point e for the network of mesh size ¼. Analogously, $T_r'(e)$ is defined for the nested network of mesh size $h' = 1/8$ in Figure 2

$$T_r'(e) = T(a') + T(b') + T(c') + T(d') - 4T(e) \quad (21)$$

and $T_r''(e)$ for the innermost nested network of mesh size $h'' = 1/16$. Our computation for the case $T(5) = 1$ gave

$$T_r(e) = -0.00301, \quad T_r'(e) = -0.000188, \quad T_r''(e) = -0.0000117 \quad (22)$$

which shows that the residual temperatures are not zero.

The discrepancy between Equation (20) and the results in (22) is traced back to the error inherent in Equation (20). Rewriting Equation (2) in the corresponding rectangular coordinates, and expanding $T(e)$ in Taylor's series, one finds (*1c*)

$$0 \equiv \left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}\right)_e = h^{-2}\,[T(a) + T(b) + T(c) + T(d) - 4T(e)] + O(h^2). \quad (23)$$

Consequently, substitution of Equation (2) with Equation (20) incurs an error of $O(h^4)$; Equation (20) is exact only $h \to 0$.

We confirm, since $h^4 = 0.00391$, that $T_r(e)$ is in the order of h^4. Furthermore, since (22) gives $T_r(e)/T_r'(e) = 16.0$ and $T_r'(e)/T_r''(e) = 16.0$, these results are in conformity with $(h/h')^4 = 16$. In conclusion, the present solution is accurate enough to detect the discretization errors of the finite-difference equations (*11*).

ACKNOWLEDGMENT

The author is indebted to E. R. Conway and B. L. Baker for their inspiration. The computer work was carried out at the University of Cincinnati Computing Center under Project 1437, where the assistance of K. C. Young is gratefully acknowledged.

NOTATION

$A_0, A_k, B_0, B_k, C_0, D_0, D_k, E_k$ = arbitrary constants, dimensionless
a = half the length of a side of the square column, L
C_n = function coefficients reduced from C_n^* by Equation (11), dimensionless
C_n^* = function coefficients determined with a line heat-source, dimensionless
h = mesh width in the unit of a, L
k = separation constant, eigenvalue, dimensionless
k_t = thermal conductivity of the square column, $ML/(Tt^3)$
N = number of retained terms in the series solution, dimensionless
p = an arbitrary point in the field, dimensionless
q = rate of heat loss per unit length of the column, ML^2/t^3
R = dimensionless radial coordinate, r/a, of an interior point
R_θ = dimensionless radial coordinate of a boundary point as a function of θ
r = radial variable of the polar coordinates, L
r_c = radius of the heating cylinder, L
r_p = distance of a point p from point A in Figure 2, L
r_δ = fictitious radius of the line heat source, L
T = dimensionless temperature defined by Equation (1)
T_N^* = dimensionless temperature determined with a set of N C_n^*'s
T_r = dimensionless residual temperature defined by Equation (20)
t_a = steady uniform temperature at the heating-cylinder surface, T
t_b = steady uniform temperature at the column walls, T
α = angle BO-15 in Figure 2, θ
δ = Dirac delta function, dimensionless
θ = angular variable of the polar coordinates, θ
ϕ = angle OAp in Figure 2, θ

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Manuscript received September 9, 1967; revision received July 29, 1968; paper accepted July 31, 1968.